Journal of Engineering Physics and Thermophysics, Vol. 70, No. 2, 1997

ON MODULATION OF SOUND BY SOUND

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UDC 534(03):534.6:536.2

Frequency shifts of continuous acoustic waves propagating in a medium which arise due to the effect of external acoustic disturbances arbitrarily propagating in the same medium are analyzed. An analytical dependence of the magnitude of the acoustic shift on the angle between the wave vectors of the probing and external waves is obtained. A corresponding directivity pattern is calculated.

It is known that in real media the principle of superposition for acoustic waves is approximate [1]. In [2] we estimated the deviation from this principle for some liquid and gaseous media. Thus, by sending sound or ultrasound waves into a medium, receiving them, and distinguishing the frequency shifts of the received oscillations, one can obtain information about external sources of acoustic signals. An ultrasound beam can act as an acoustic receiving antenna and a microphone. If the ultrasound beam is affected by an acoustic field from some standard source, then valuable information can be obtained about the nonlinear properties of the medium in which the measurements are made, because the nonlinear properties of the medium determine the deviation from the principle of superposition. To analyze these possibilities one should study in more detail the interaction of acoustic waves propagating arbitrarily to each other. In [2], only the simplest case in which the directions of the wave vectors of two acoustic plane waves coincide is considered.

Let continuous plane acoustic waves propagate in the direction of the x axis. For certainty we assume that the emitter is at $x = 0$, the receiver is at $x = L$, and the frequency of the emitted waves is constant and equal to f_0 (probing radiation). Simultaneously, plane acoustic waves with frequency F (external radiation) propagate in the medium; their direction makes an angle α with the x axis.

The solution method, as in [2], is as follows. First we find the law of probing wave motion in a medium disturbed by an external acoustic field. It is easily shown that this law is a solution of the differential equation

$$\frac{dx}{d\tau} = v_0 + \Delta v_0 \sin \Omega \left(\tau - \frac{x \cos \alpha}{v_0} + t\right) \tag{1}$$

under the corresponding initial condition. Without loss of generality, the initial condition can be taken in the form

$$x|_{\tau=0} = 0 \,. \tag{2}$$

Here $\Omega = 2\pi F$, $\varphi = \Omega t$ is the initial phase of external acoustic field oscillations at the point $x = 0$ at the moment of transmission of probing wave radiation (i.e., at $\tau = 0$).

The solution of Eq. (1) which satisfies condition (2) has the form

$$\arctan \left[A \frac{\tan \frac{\Omega}{2} \left(\tau - \frac{x \cos \alpha}{v_0} + t\right) - 1}{\tan \frac{\Omega}{2} \left(\tau - \frac{x \cos \alpha}{v_0} + t\right) + 1} \right] - \arctan \left[A \frac{\tan \frac{\Omega}{2} t - 1}{\tan \frac{\Omega}{2} t + 1} \right] =$$

Academic Scientific Complex "A. V. Luikov Heat and Mass Transfer Institute of the Academy of Sciences of Belarus," Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 2, pp. 230-235, March-April, 1997. Original article submitted April 26, 1995.

$$= \frac{\Omega\tau}{2v_0}\sqrt{v_0^2(1-\cos\alpha)^2 - \Delta v_0^2\cos^2\alpha}\,, \quad \alpha_0 < \alpha < 2\pi - \alpha_0 \tag{3}$$

and

$$\ln\left|\frac{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}\left(\tau - \frac{x\cos\alpha}{v_0} + t\right) - \Delta v_0\cos\alpha - \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}\left(\tau - \frac{x\cos\alpha}{v_0} + t\right) - \Delta v_0\cos\alpha + \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}\right| -$$

$$- \ln\left|\frac{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}t - \Delta v_0\cos\alpha - \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}t - \Delta v_0\cos\alpha + \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}\right| =$$

$$= \frac{\Omega}{v_0}\sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}\,\tau\,, \quad -\alpha_0 \le \alpha \le \alpha_0\,, \tag{4}$$

where

$$A = \sqrt{\left(\frac{v_0(1-\cos\alpha) + \Delta v_0\cos\alpha}{v_0(1-\cos\alpha) - \Delta v_0\cos\alpha}\right)}\,, \quad \alpha_0 = \arccos\frac{v_0}{v_0 + \Delta v_0}\,. \tag{5}$$

If in expressions (3) or (4) it is assumed that $x = L$, then from them one can find the time $\tau = \tau_d$ in which a probing wave reaches the receiver, i.e., the delay time. At constant L, v_0, Δv_0, and Ω τ_d will be the function of $\varphi = \Omega t$, i.e., t can be considered as a current time related to the emitter.

For convenience instead of τ_d we introduce the quantity $\tau_d = L/v_0 + \Delta\tau_d$. Then we have instead of (3) and (4)

$$\arctan\left[A\,\frac{\tan\frac{\Omega}{2}\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right) - 1}{\tan\frac{\Omega}{2}\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right) + 1}\right] - \arctan\left[A\,\frac{\tan\frac{\Omega}{2}t - 1}{\tan\frac{\Omega}{2}t + 1}\right] -$$

$$- \left(\frac{L}{v_0} + \Delta\tau_d\right)\frac{\Omega}{2v_0}\sqrt{v_0^2(1-\cos\alpha)^2 - \Delta v_0^2\cos^2\alpha} = 0\,, \quad \alpha_0 < \alpha < 2\pi - \alpha_0\,; \tag{6}$$

$$\ln\left|\frac{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right) - \Delta v_0\cos\alpha - \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right) - \Delta v_0\cos\alpha + \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}\right| -$$

$$- \ln\left|\frac{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}t - \Delta v_0\cos\alpha - \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}{v_0(1-\cos\alpha)\tan\frac{\Omega}{2}t - \Delta v_0\cos\alpha + \sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2}}\right| -$$

$$- \left(\frac{L}{v_0} + \Delta\tau_d\right)\frac{\Omega}{v_0}\sqrt{\Delta v_0^2\cos^2\alpha - v_0^2(1-\cos\alpha)^2} = 0\,, \quad -\alpha_0 \le \alpha \le \alpha_0\,. \tag{7}$$

We find the shift of the frequency f of the received probing waves relative to the frequency f_0 of the emitted waves, i.e., $\Delta f = f - f_0$. It can be shown that for Δf the relation

$$\Delta f(t) = -f_0 \frac{d\tau_d}{dt} \equiv -f_0 \frac{d(\Delta\tau_d)}{dt}. \tag{8}$$

is valid with sufficient accuracy (the error is of the order of $\Delta f/f_0$ compared to unity).

The value of $d(\Delta\tau_d/dt)$ can be found as the derivative of the implicit function which is the left-hand side of (6) or (7). After differentiation we obtain

$$\Delta f(t, \alpha) = f_0 \frac{\Delta v_0}{v_0} \frac{\sin\left[\Omega\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right)\right] - \sin\Omega t}{\left\{1 + \frac{\Delta v_0}{v_0}\sin\left[\Omega\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + t\right)\right]\right\}\left(1 - \cos\alpha - \frac{\Delta v_0}{v_0}\cos\alpha\sin\Omega t\right)}. \tag{9}$$

This formula turns out to be valid for any values of α. The second term in the braces of the denominator can be ignored, since $\Delta v_0/v_0 \ll 1$ (the term in the brackets cannot be ignored, because the entire expression could vanish). Thus,

$$\Delta f(\tau, \alpha) = f_0 \frac{\Delta v_0}{v_0} \frac{\sin\left[\Omega\left(\Delta\tau_d + \frac{L(1-\cos\alpha)}{v_0} + \tau\right)\right] - \sin\Omega\tau}{1 - \cos\alpha - \frac{\Delta v_0}{v_0}\cos\alpha\sin\Omega\tau}. \tag{10}$$

In the latter expression t is replaced by the previously adopted τ for the current time. As is seen from (10), $\Delta f(\tau, \alpha)$ is a periodic function of time with period $2\pi/\Omega = 1/F$. This function attains its greatest value at $\alpha = 0$, which for $\tau \to 0$ is equal to

$$\Delta f_{\max}(0) = f_0 \frac{\Omega \Delta v_0 L}{v_0^2} = 2\pi f_0 \frac{\Delta v_0}{v_0} \frac{L}{\lambda}. \tag{11}$$

We denote the peak (maximum in absolute value) value of $\Delta f(\tau, \alpha)$ as calculated by (1) with allowance for equations (6) and (7) as $\Delta f_{\max}(\alpha)$. To obtain the directivity pattern of $\Delta f_{\max}(\alpha)$ we performed calculations on a computer. Here the time interval $\Delta\tau = 1/F$ was divided to $n = 20$ equal parts. At the boundaries of these parts the value of $\Delta\tau_d$ was found by a numerical solution of equation (6) or (7), depending on the chosen value of the angle α. The angle α was split into 10-degree intervals from 0 to π. The frequency of probing ultrasound waves was $f_0 = 10^6$ Hz, the studied medium was water, and $v_0 = 1500$ m/sec. According to the data of [3], for water the velocity of ultrasound propagation changes by $\Delta v_0 = 1.84 \cdot 10^{-6}$ m/sec as pressure changed by 1 Pa. For the external acoustic field the frequency values were $F = 10^4$ and 10^5 Hz, the length of the acoustic wave was $L = 0.75$ and $L = 1.5$ m; thus, calculations wereperformed for values of the ratio L/λ equal to 10, 50, and 100. The calculation by formula (11) for these values of L/λ, under the condition that the sound pressure produced by the acoustic field is 1 Pa, gives values of the maximum shift of the frequency of probing waves $\Delta f_{\max}(0)$ equal to $7.7 \cdot 10^{-2}$, $3.85 \cdot 10^{-1}$, and $7.7 \cdot 10^{-1}$ Hz, respectively, which is easily measured by modern devices. The calculated data for other values of the angle are presented in Table 1.

The overall character of the dependence $\Delta f_{\max}(\alpha)$ is similar for different values of the ratio L/λ. The directivity pattern is strongly extended toward the polar axis $\alpha = 0$; the moreso, the greater the value of L/λ. For example, for $L/\lambda = 100$, $\Delta f_{\max}(\alpha)$ is by two orders of magnitude smaller at $\alpha = 45^\circ$ than at $\alpha = 0$. Table 2 presents the values of angles α at which for the chosen values of L/λ $\Delta f_{\max}(\alpha)$ decreases by factors of 2, 3, and 10 compared to $\Delta f_{\max}(0)$. These data show that to solve problems of direction finding for an external source of acoustic signals, one should select the maximum possible base L for the probing beam.

TABLE 1. Angular Distribution of Relative Peak Value of Frequency Shift $\Delta f_{max}(\alpha)/\Delta f_{max}(0)$

α	$F = 10^4$ Hz, $L/\lambda = 10$	$F = 10^5$ Hz, $L/\lambda = 50$	$F = 10^5$ Hz, $L/\lambda = 100$
0	1	1	1
5	–	–	$6.83 \cdot 10^{-1}$
8	–	$5.12 \cdot 10^{-1}$	$2.97 \cdot 10^{-1}$
10	–	$4.16 \cdot 10^{-1}$	$1.17 \cdot 10^{-1}$
15	$8.61 \cdot 10^{-1}$	$1.79 \cdot 10^{-1}$	$7.85 \cdot 10^{-2}$
20	$4.24 \cdot 10^{-1}$	$8.77 \cdot 10^{-2}$	$5.08 \cdot 10^{-2}$
30	$2.19 \cdot 10^{-1}$	$4.30 \cdot 10^{-2}$	$2.15 \cdot 10^{-2}$
40	$1.26 \cdot 10^{-1}$	$2.53 \cdot 10^{-2}$	$1.26 \cdot 10^{-2}$
50	$6.41 \cdot 10^{-2}$	$1.77 \cdot 10^{-2}$	$8.83 \cdot 10^{-3}$
60	$4.95 \cdot 10^{-2}$	$9.90 \cdot 10^{-3}$	$4.95 \cdot 10^{-3}$
70	$3.58 \cdot 10^{-2}$	$9.45 \cdot 10^{-3}$	$4.73 \cdot 10^{-3}$
80	$2.86 \cdot 10^{-2}$	$6.75 \cdot 10^{-3}$	$3.37 \cdot 10^{-3}$
90	$1.62 \cdot 10^{-2}$	$3.25 \cdot 10^{-3}$	$1.62 \cdot 10^{-3}$
100	$2.71 \cdot 10^{-2}$	$5.28 \cdot 10^{-3}$	$2.64 \cdot 10^{-3}$
110	$2.36 \cdot 10^{-2}$	$2.10 \cdot 10^{-3}$	$1.05 \cdot 10^{-3}$
120	$1.67 \cdot 10^{-2}$	$3.34 \cdot 10^{-3}$	$1.67 \cdot 10^{-3}$
130	$1.93 \cdot 10^{-2}$	$2.83 \cdot 10^{-3}$	$1.42 \cdot 10^{-3}$
140	$1.67 \cdot 10^{-2}$	$2.90 \cdot 10^{-3}$	$1.45 \cdot 10^{-3}$
150	$1.58 \cdot 10^{-2}$	$2.74 \cdot 10^{-3}$	$1.37 \cdot 10^{-3}$
160	$1.62 \cdot 10^{-2}$	$2.20 \cdot 10^{-3}$	$1.10 \cdot 10^{-3}$
170	$1.50 \cdot 10^{-2}$	$2.99 \cdot 10^{-3}$	$1.50 \cdot 10^{-3}$
179	$9.07 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$9.71 \cdot 10^{-4}$

TABLE 2. Some Geometric Characteristics of the Directivity Pattern

F, Hz	L/λ	Values of α at different $\Delta f_{max}(0)/\Delta f_{max}(\alpha)$		
		2	3	10
10^4	10	18	25	40
10^5	50	9	12	19
10^5	100	6	8	12

As is seen from the figure, the dependence $\Delta f_{max}(\alpha)$ has a number of maxima and minima; in what follows we shall show that their total number is equal to an integer from the ratio $2L/\lambda$ for each quarter of the total circle. The values of angle α_i that correspond to minima and maxima of the dependence $\Delta f_{max}(\alpha)$ can be approximately obtained by equating to zero the derivative of the function $\Delta f(\tau, \alpha)$ with respect to α, which is expressed by formula (10) with allowance for relation (6), from which the expression for $d(\Delta\tau_d)/d\alpha$ when $\alpha > \alpha_0$ is found. If we perform this procedure, we obtain extremely cumbersome expressions, which are not presented here. Therefore, we adopt one more simplification: we assume that all maxima and minima of the function $\Delta f_{max}(\alpha)$ occur at angles α_i such that $\alpha_i \gg \alpha_0$. The latter inequality is fulfilled for the adopted values of ν_0 and $\Delta\nu_0$ $\alpha_0 = \arccos[\nu_0/(\nu_0 + \Delta\nu_0)] \approx 10^{-9}$ rad, and the angle α, which corresponds to the first maximum (after the zeroth at $\alpha = 0$),

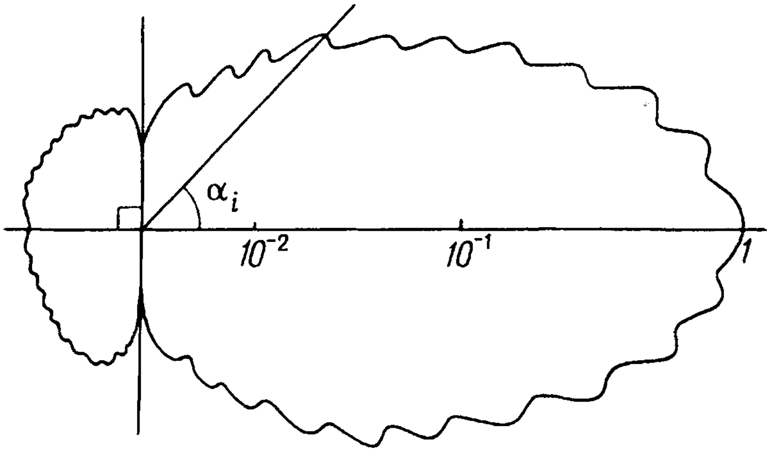

Fig. 1. Diagram of angular dependence of the relative peak value of the frequency shift $\Delta f_{\max}(\alpha)/\Delta f_{\max}(0)$ for $L/\lambda = 10$ in logarithmic scale.

proceeding from geometric considerations, is of the order of $\pi/2n$, where n is the integer part of the ratio L/λ. For example, $\alpha_1 \approx 1.6 \cdot 10^{-2}$, for $L/\lambda = 100$ so that $\alpha_0/\alpha_1 = 6 \cdot 10^{-8} \ll 1$.

With this simplification it is true (see relations (3)-(5)) that $\Delta\nu_0 \cos\alpha \ll \nu_0(1 - \cos\alpha)$, $A \approx 1$, $dA/d\alpha \approx 0$, and one can show that the approximate relation

$$\frac{d\,(\Delta\tau_{\rm d})}{d\alpha} \approx \Delta\tau_{\rm d} \tan\alpha \,. \tag{12}$$

is fulfilled with a sufficient degree of accuracy.

Allowing for the latter relation and the adopted simplifications, after differentiation we have from relation (10)

$$\frac{d\,[\Delta f\,(\tau, \alpha)\,]}{d\alpha} \approx f_0 \frac{\Delta\nu_0}{\nu_0} \, \frac{\sin\alpha \left\{\cos k\,(\tau, \alpha) \left[\Omega \left(\dfrac{\Delta\tau_{\rm d}}{\cos\alpha} + \dfrac{L}{\nu_0}\right)\right] - \sin k\,(\tau, \alpha)\right\}}{\left[1 - \cos\alpha \left(1 + \dfrac{\Delta\nu_0}{\nu_0} \sin k\,(\tau,\alpha)\right)\right]^2} , \tag{13}$$

where $k(\tau, \alpha) = \Omega(\Delta t_{\rm d} + L(1 - \cos\alpha)/\nu_0 + \tau)$. The error of the latter equality (the difference from the accurate one) is of the order of $\Delta\nu_0/\nu_0$. Equating (13) to zero we obtain two equations with respect to α

$$\sin\alpha = 0 \,, \tag{14}$$

$$\tan\left[\Omega\left(\Delta\tau_{\rm d} + \frac{L\,(1 - \cos\alpha)}{\nu_0} + \tau\right)\right] - \Omega\left(\frac{\Delta\tau_{\rm d}}{\cos\alpha} + \frac{L}{\nu_0}\right) = 0 \,. \tag{15}$$

Equation (14) gives the value of the angle $\alpha = 0; \pi$, which corresponds to the zeroth maximum. If we above from $\Delta f(\tau, \alpha)$ to $\Delta f_{\max}(\alpha)$, for the derivative $d[\Delta f_{\max}(\alpha)]/d\alpha$ we obtain the same expression (13); but in this expression and, correspondingly, in (15), the quantity τ becomes not an independent variable but some function of the angle α. The number of extrema of the function $\Delta f_{\max}(\alpha)$ within the range $[0, \pi]$ will be equal to the number of roots of transcendental equation (15) within the same range and to the number of roots of the simpler transcendental equation

$$\tan\frac{\pi}{2} k\,(1 - z) - k\frac{\pi}{2} = \frac{\Omega\Delta\tau_{\rm d}}{z} \,, \tag{16}$$

where $k = (2/\pi)(\Omega L/v_0) = 4L/\lambda$ within the range of $z = \cos\alpha$ variation is equal to $[-1, 1]$. Allowing for the fact that $\Omega\Delta\tau_d \leq 2\pi$, one can find, for example, graphically, that for large k the number of roots is equal to k. Consequently, the number of extrema of the function $\Delta f_{max}(\alpha)$ within the range of α variation of from 0 to π is equal to $4L/\lambda$. Since the function $\Delta f_{max}(\alpha)$ is continuous, the maxima and minima should alternate; thus, within the range $[0, \pi]$ it has $2L/\lambda$ maxima and the same number of minima. As a result we have L/λ "lobes" of the directivity pattern in each quarter of the circle. In this case, as calculation shows, the function $\Delta f_{max}(\alpha)$ does not become zero between the "lobes"; it only takes some minimum value which differs from zero. It is seen from Eq. (16) that it has a singularity $\alpha = \pi/2$ at which the derivative $d[\Delta f_{max}(\alpha)]/d\alpha$ has a discontinuity and changes sign; this is a cusp of the first order for the function $\Delta f_{max}(\alpha)$. In this case the value of $\Delta f_{max}(\pi/2)$ does not vanish but has a finite value of $1.25 \cdot 10^{-3}$ Hz for the three chosen values of the ratio L/λ.

As is seen from Table 1, the depth of modulation of a 1-MHz probing signal at a sound pressure of 1 Pa produced in water by a source of acoustic disturbances is fractions of Hertz at small angles α. Modern methods and engineering means make it possible to measure and register such modulation. Consequently, practical use of the described method in hydroacoustics and also, as was shown above, for precision studies of nonlinear properties of liquid media is possible in principle. Similar calculations for gaseous media show also the possibility of practical use of the method. Although in this case one should use lower frequencies of probing ultrasound due to strong attenuation in gases.

NOTATION

x, coordinate; L, length of acoustic base; f_0 and F, frequencies of probing and external acoustic waves, respectively; α, angle; τ and t, time; v_0 and Δv_0, velocity of propagation of ultrasound waves in undisturbed media and its maximum change under the effect of an external acoustic field; $\lambda = v_0/F$.

REFERENCES

1. M. A. Isakovich, in: General Acoustics [in Russian], Moscow (1973), pp. 29, 46-47.
2. V. I. Krylovich, Inzh.-Fiz. Zh., 43, No. 5, 826-833 (1982).
3. L. Bergman, Ultrasound and Its Application in Science and Technology [Russian translation], Moscow (1956).